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(Residential Autonomous College affiliated to University of Calcutta)

B.A./B.Sc. FIRST SEMESTER EXAMINATION, DECEMBER 2018

FIRST YEAR [BATCH 2018-21]

Date : 24/12/2018 **MATHEMATICS FOR ECONOMICS [General]**

Time : 11 am – 2 pm **Paper : I** Full Marks : 75

[Use a separate Answer Book for <u>each Group</u>]

<u>Group – A</u>

Answer <u>any five</u> questions from <u>Question Nos. 1 to 8</u> : [5×7]

1. a) Show that the set $S=\left\{x\in\mathbb{R}\,\middle|\,x^2-24x+119\le 0 \text{ and } x\ne 7\right\}$ is neither open nor closed in $\mathbb{R}$. [4]

   b) Consider the set $A_k=\left\{x\,\middle|\,x=kn \text{ and } n\in\mathbb{N}\right\}$ where $\mathbb{N}=\{1,2,......\}$ is the set of natural numbers. Write down the set A_2 and A_3 then show that $A_2\cap A_3=A_6$ [3]

2. a) Prove that $\lim\limits_{n\to\infty}\left(\frac{1}{\sqrt{n^2+1}}+\frac{1}{\sqrt{n^2+2}}+.......+\frac{1}{\sqrt{n^2+n}}\right)=1$ [5]

   b) Give example of an oscillatory sequence with infinite oscillation. [2]

3. a) Show that the sequence $x_{n+1}=\frac{1}{2}\left(x_n+\frac{2}{x_n}\right)$ for $n\ge 2$ and $x_1=2$ converges to $\sqrt{2}$. [5]

   b) Using part (a), give a sequence which converges to $\sqrt{\alpha}$ for $\alpha>0$. [2]

4. a) State Archimedean Property of $\mathbb{R}$.

   b) Prove that the series $\left(\frac{1}{2}\right)^p+\left(\frac{1\cdot3}{2\cdot4}\right)^p+\left(\frac{1\cdot3\cdot5}{2\cdot4\cdot6}\right)^p+.........$ is convergent for $p>2$ and divergent for $p\le 2$. [2+5]

5. a) Prove that the series $\sum\limits_{n=1}^{\infty}\frac{1}{n}$ is divergent. [5]

   b) Write down the Cauchy's principle of convergence for a series $\sum\limits_{n=1}^{\infty}u_n$. [2]

6. Verify whether the following functions are onto, one-to-one or bijection.

   a) $f:\mathbb{R}\to\mathbb{R}$ such that $f(x)=x^2$

   b) $f:\mathbb{R}_+\to\mathbb{R}$ such that $f(x)=x^2$

   c) $f:\mathbb{R}\to\mathbb{R}_+\cup\{0\}$ such that $f(x)=x^2$

   d) $f:\mathbb{R}_+\to\mathbb{R}_+$ such that $f(x)=x^2$

   Here, $\mathbb{R}_+$ denotes the set of all positive real numbers. [2+2+2+1]

7. a) Represent the set of ordered pairs $S=\{(x,y) \mid |y-1|-x\le 1, -\infty<x,y<\infty\}$, in a graph. Is it a closed or an open set? [4]

b) Draw the functions $f(x)=\text{Min}\{|x|,1\}$ for $-\infty<x<\infty$. Comment on the continuity and the differentiability of f at $x=+1$ and $x=-1$. [3]

8. a) Check whether the sequence $s_n=\{(-1)^n\}_{n=1}^{\infty}$ is convergent or not. Consider a sequence $s_n\in\left(a-\frac{1}{n},a+\frac{1}{n}\right)$ for $n\in\mathbb{N}$, $a\in\mathbb{R}$ where $\mathbb{N}$ and $\mathbb{R}$ are respectively the set of natural numbers and the set of real numbers. Show that $s_n\to a$ as $n\to\infty$. [2+2]

b) If $A=\{a_1,a_2,a_3\}$ and $B=\{b_1,b_2\}$, then how many different functions $F:A\to B$ are possible? [3]

Group – B

***Answer any five questions from Question Nos. 9 to 16* :** [5×6]

9. a) Solve the equation $x^3+8=0$. [2]

b) If α,β are the roots of the equation $t^2+2t+4=0$ and m is a positive integer, then prove that

$$\alpha^m+\beta^m=2^{m+1}Cos\frac{2m\pi}{3}.$$ [4]

10. a) Let $(G,\circ)$ be a group . A relation ρ on G is defined by "a ρ b if and only if $b=g\circ a\circ g^{-1}$ for some g in G; $a,b\in G$". Prove that ρ is an equivalence relation.

b) Prove that the set of complex numbers of unit modulus forms a commutative group with respect to multiplication . [3+3]

11. a) Prove that in a group $(G,\circ)$, $(a\circ b)^{-1}=b^{-1}\circ a^{-1}$ for all $a,b\in G$.

b) Let, addition $\oplus$ and multiplication $\odot$ be defined on the ring $(\mathbb{Z},+,.)$ by $a\oplus b=a+b-1$, $a\odot b=a+b-a.b$, $\forall\, a,b\in\mathbb{Z}$. Prove that $(\mathbb{Z},\oplus,\odot)$ is a ring with unity. [2+4]

12. a) Prove that , in a field F $a^2=b^2$ implies either $a=b$ or $a=-b$, $\forall a,b\in F$.

b) Solve by Cramer's rule:

$$x+y+z=1$$
$$ax+by+cz=k$$
$$a^2x+b^2y+c^2z=k^2 \quad , \text{ where } a\ne b\ne c \quad \text{and} \quad k\in\mathbb{R}$$ [2+4]

13. Examine whether the ring of matrices form a field under matrix addition and matrix multiplication

a) $\left\{\begin{pmatrix} a & b \\ 3b & a \end{pmatrix} : a, b \in \mathbb{Q}\right\}$

b) $\left\{\begin{pmatrix} a & b \\ 3b & a \end{pmatrix} : a, b \in \mathbb{R}\right\}$ [3+3]

14. Consider the matrix on friendship involving five individuals A_1, A_2, A_3, A_4 and A_5.

$$\begin{array}{c} \\ A_1 \\ A_2 \\ A_3 \\ A_4 \\ A_5 \end{array} \begin{array}{c} \begin{array}{ccccc} A_1 & A_2 & A_3 & A_4 & A_5 \end{array} \\ \begin{bmatrix} 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 \\ 1 & 0 & 0 & 1 & 0 \\ 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 \end{bmatrix} \end{array} = F$$

Where, $a_{ij} = \begin{cases} 1 & \text{if } A_i \text{ is friend with } A_j \\ 0 & \text{otherwise} \end{cases}$

a) Calculate F^2. [2]

b) What does each diagonal term of F^2 mean? [2]

c) What does each off-diagonal term of F^2 mean? [2]

15. a) Let $A = \begin{pmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{pmatrix}$. Show that $A^2 - 4A - 5I_3 = 0_3$,where I_3 and 0_3 are the identity and the null matrices of order 3 respectively. Hence, obtain A^{-1}.

b) Let A and B be two non-singular matrices of order $n \times n$ .Is A+B non-singular ? [4+2]

16. a) Determine the values of α, β and γ when $\begin{bmatrix} 0 & 2\beta & \gamma \\ \alpha & \beta & -\gamma \\ \alpha & -\beta & \gamma \end{bmatrix}$ is orthogonal. [4]

b) Let A be a non-singular matrix of order 4 .Determine the rank of the matrix Adj A . [2]

Answer <u>any two</u> questions from <u>Question Nos. 17 to 19</u> **:** [2×5]

17. (a) Determine the function whose first difference is $3x^2 - 5x + 7$.

(b) Eliminate A and B from $y_n = A \cdot 3^n + B \cdot 4^n$ and determine the corresponding difference equation of lowest order. [3+2]

18. Solve : $u_x - u_{x-1} + 2u_{x-2} = x + 2^x$ [5]

19. Reduce the matrix A to the fully reduced normal form and find non-singular matrices P and Q such that PAQ is the fully reduced normal form , where $A = \begin{pmatrix} 2 & 1 & 2 \\ 1 & 0 & 1 \\ 1 & 1 & 1 \end{pmatrix}$.

Also find the rank of A. [5]

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